

UNIVERSITÄT AUGSBURG

INSTITUT FÜR MATHEMATIK

Universitätsstraße 14
D-86135 Augsburg

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M. Balinski, PhD
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by

*Michel Balinski
and
Friedrich Pukelsheim*

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Computing electoral districts

Michel Balinski

CNRS and Ecole Polytechnique

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Computing Electoral Districts

Michel Balinski
CNRS and Ecole Polytechnique, Paris

Short Abstract

Political gerrymandering is the practice of dividing a geographical area into electoral districts, often of highly irregular shape, to favor one political party. New technology has turned the amateurish efforts of the past into precise gerrymandering tools that can seriously subvert an electorate's intent (and have). This talk will explain why, and how, and what can be done about it.

Extended abstract

On April 28, 2004 the United States Supreme Court announced its decision in the case of *Vieth v. Jubelirer*. It refused to declare unconstitutional Pennsylvania's new map of 19 congressional districts (based on the census of 2000, first used in the 2002 elections). Its bizarrely shaped districts have been compared to animals: the "supine seahorse" and the "upside-down Chinese dragon." By all accounts Pennsylvania slightly favors the Democratic Party. In the elections of 2000 (when the state had 21 seats) it elected 11 Republicans and 10 Democrats (while giving Gore 50.6% and Bush 46.4% in the state-wide vote); in 2002, with the new map, it elected 12 Republicans and 7 Democrats (while giving a Democratic gubernatorial candidate 55% in the state-wide vote). Five Republican candidates ran unopposed, one Democratic candidate ran unopposed. The map "kidnapped" three Democratic incumbents; it systematically "cracked" Democratic voters among districts; and "packed" and "stacked" them into districts. Yet the map is "perfect": every district has either 646,371 or 646,372 inhabitants. Not one Justice denied that the map is a blatant *political gerrymander* ("the practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage by diluting the opposition's voting strength").

Texas, with its new congressional map, elected 17 Democratic and 15 Republican Representatives in 2002, and also gave absolute political control of the State government to the Republicans. The new State government immediately redrew the map of the congressional districts. It is widely agreed that in the 2004 elections Texas will elect 10 Democrats (perhaps 9) and 22 (perhaps 23) Republicans. Yet every district has either 651,619 or 651,620 inhabitants. Gerrymandering is perfectly ecumenical: the Democrats believe in it as much as their Republican brethren. In Maryland the map drawn by the Democrats changed what had been an equal division of the seats into a 6 to 2 split to their advantage and deliberately eliminated a targeted incumbent. In Georgia the Democrat's map increased their congressional delegation by two and successfully "kidnapped" a manager of the effort to impeach President Clinton (despite the loss of one Senator and the governorship in state-wide races). California's map is bipartisan ... yet suspect: 50 of 53 Representatives including every incumbent candidate was elected with at least 60% of the votes in 2002.

These instances, and others, have provoked a public outcry: *district maps determine the winners, not elections!*

Why and how has this happened? First, Supreme Court decisions have set precedents that are confused and often contradictory. Second, new computer technology has permitted maps and their political implications to be defined easily and quickly. Third, the increasingly predictable behavior of voters has made the political implications of district lines more accurate.

In the opinion of the plurality (four of nine) in the case of *Vieth v. Jubelirer*, claims of partisan gerrymandering should be "nonjusticiable because no judicially discernible and manageable standards for adjudicating such claims exist." Their reason (if not their reasoning) is sound: there is no theory or body of knowledge capable of distinguishing which of two district maps is "fairer." *If single-member constituencies are to remain in use (and they exist in many countries) then a central problem in the mathematics of political systems is to develop rigorous criteria for "fair districting" ... or to show that it is impossible to do so.*

The U.S. Supreme Court's ruling and opinion will no doubt open the floodgates, provoking new gerrymanders wherever one of the two parties has absolute political control of the State. A swing of as much as 5% of the vote from the Republicans to the Democrats is already expected to result in practically no increase of Democratic representation. A major crisis threatens the democratic institutions of the nation.

The judicial system has proved itself incapable of providing relief. Congress has the constitutional right to impose a fair electoral system. *What should the Congress do?* The problem is at once urgent and important.

This talk will outline the problem in the context of the current situation in the United States, and will advance several tentative answers.

1. Why the problem is urgent and important

The French Assemblée Nationale :

Apportionment and districting date from 1986 (based on census of 1982). Based on 1999 census :

	population	députés
Haute-Garonne	1 046 338	8
Moselle	1 023 447	10

(47 such “reversals”)

	average population	inequality
25 most populated départements (>50% pop)	112 123	41.9%
25 least populated départements	79 043	

	Population	inequality
2 ^e circ. Lozère	34 374	448%
2 ^e circ. Val-de-Marne	188 200	

Within le Var :

	Population inequality	
2 ^e circonscription	73 946	144%
6 ^e circonscription	180 153	

The US House of Representatives :

2002 election (435 Representatives) :

- 386 candidates were incumbents : exactly 4 defeated by outsiders
- 81 candidates unopposed
- 338 incumbents re-elected with >60% of votes
- Representatives :
 - <10% elected with <60% of votes

Candidates state-wide (“natural districts”) :

50% elected with <60% of votes

- In California : 50 (of 53) candidates elected with >60% of votes
- Widely believed that 400 of the 435 seats are “safe”

2000 election :

Gore : 50 999 897 Bush : 50 456 002

Suppose the Electoral College gave 1 vote to that candidate having the most votes in each congressional district (as defined in 2002). The outcome would have been :

Gore : 198 Bush : 237

Their proportional shares :

Gore : 219 Bush : 216

Gerrymandering Pennsylvania :

Political gerrymandering is “the practice of dividing a geographical area into electoral districts, often of highly irregular shape, to give one political party an unfair advantage by diluting the opposition’s voting strength.” (*Black’s Law Dictionary* (1999))

Population : 12 291 054

Congressional districts : 19 (down from 19)

Counties : 67

Voting precincts : 9 427

Census tracts : 322 424 (avg. persons/tract = 38)

First district plan :

A frank and open gerrymander for the Republicans

3 Democratic incumbents “kidnapped”

Democrats systematically “cracked” and “packed”

84 local governments split (25 counties and 59 cities, boroughs or townships)

6 voting precincts split

Montgomery County : split 6 ways

Largest district : 646 380

Smallest district : 646 361

Result in 2002 elections :

Republicans: 12 (before 11)

Democrats : 7 (before 10)

Democrats win in state-wide races

Court case *Vieth v. Jubelirer* in Federal District

Court :

No to charge of partisan gerrymandering

Yes to charge not as equal as possible

Second district plan :

Extremely minor adjustments – new districts
contain 99.34% of inhabitants of old districts
110 local governments split (29 counties and 81
cities, boroughs or townships)

Largest district : 646 372

Smallest district : 646 371

1 person difference : How was this done ! ? :

“Caliper’s Maptitude for Redistricting”

Technology has created a fundamental change in the practice of democracy : it may well be **impossible** to devise a set of criteria for determining what is a fair districting plan and what is not.

Districting determines winners, not voters !

2. Why the impasse ?

In France :

Momentum, cynicism, politics, an excessively shy Conseil Constitutionnel. The law calls for a reapportionment and redistricting after every second census : today's Assemblée determined on the basis of census of 1982, yet censuses conducted in 1990 and 1999.

In the United States :

History; and US Supreme Court decisions that are confused and often contradictory.

Judge Felix Frankfurter's statement (1946), "Courts ought not to enter this political thicket," prevailed until the spread between rural over-representation and urban under-representation became too blatant, and the Kennedy administration supported reform.

Baker v. Carr (1962). Tennessee legislature :

2	340	person	district	elected	2	representatives
25	316	--	--	--	2	--
312	345	--	--	--	7	--

The Court gave little guidance, but opened the floodgates of litigation attacking situations. Chief Justice Earl Warren called it “the most important case [of my] tenure on the Court.”

Wesberry v. Sanders, February 1964. “... as nearly as is practicable one man’s vote in a congressional election is to be worth as much as another’s.”

Kirkpatrick v. Preisler, April 1969. “ ...the ‘as nearly as practicable’ standard requires that the State make a good faith effort to achieve precise mathematical equality.”

Karcher v. Daggett, June 1983. New Jersey’s redistricting did not meet the standard :

Largest : 527 472	Smallest : 523 798
Inequality : 0.7%	

Vieth v. Jubelirer, April 2002, District Court : a difference of 19 persons does not meet it either.

Other historical forces were at work too.

The Voting Rights Act of 1965 finally assured Blacks the legal possibility of voting ... but not the possibility of being elected themselves.

In 1982 it was reinforced : electoral procedures could not dilute the votes of minorities defined by *race, color or language*. This allowed the creation of “majority-minority” districts ... and promoted the “unholy alliance” between Black Democrats and White Republicans (and the Republican successes in the 1992 elections) ... and very strange districts indeed !

Davis v. Bandemer, June 1986. A *cohesive* political group – like a racial minority – could be protected from a dilution of its votes, but *actual* and *intended* discrimination had to be established, *and* the criterion “seats in proportion to votes” is *not* a constitutional principle.

These and other cases were typically controversial and decided by close votes.

Throughout, members of the Supreme Court (in majority and minority opinions) tussled with **criteria** or **standards** by which to judge when a district plan is constitutional and when not ... other than the obvious “good faith effort to achieve precise mathematical equality” among the populations of the districts.

Districting plans should (for some judges, sometimes should not) :

- have connected districts
- have compact districts
- have frontiers agreeable to the eye
- conform to traditional political, or administrative frontiers
- respect communities of common interest
- protect incumbents
- be judged not as a total plan but on the basis of single districts
- be judged by the process by which they were devised
- not dilute the votes of identifiable minorities or of identifiable cohesive political groups

- not be evaluated on the basis of “seats in proportion to votes” of any group, which is not a constitutional principle, so not germane to determining dilution – such judgements depend on “the totality of the circumstances”

But while “segregating voters on the basis of race is not a lawful one, ... the fact [is] that partisan districting is a lawful and common practice”

Vieth v. Jubelirer, April 2004, Supreme Court. Four of the majority of five judges that decided *Bush v. Gore* in December 2000 signed the plurality decision written by A. Scalia :

“Eighteen years of essentially pointless litigation have persuaded us that *Bandemer* is incapable of principled application. We would therefore overrule that case, and decline to adjudicate these political gerrymandering claims.”

The fifth agreed ... but hoped criteria will eventually be found.

The judicial system has failed.

In the United States of today there is nothing that can stop the legislature of a state to district in any way they like ... so long as the populations of the districts are as nearly equal as possible.

Texas 2002. Redistricted in 2000, results in the 2002 election :

Democrats : 17

Republicans : 15

In elections of 2002 : both Houses and the governorship of the State became Republican. They have redistricted.

Predicted result in 2004 :

Democrats : 10 (or 9)

Republicans : 22 (or 23)

3. What can be done

The districting problem

- A *map* of indecomposable units, called
- *cantons* (in France ; or communes, municipalities, precincts ...),
- their *populations*, and
- the number n of districts in the département (state or region).

A *district* is a subset of cantons that constitute a *connected* region.

Imagine a jigsaw puzzle, one piece for each canton, whose “weight” is the population of the canton.

The problem is to assemble the pieces into n separate sets of connected pieces that are “as nearly as possible of equal” total weight.

This is the problem as defined by the French electoral law of 1986 (still standing ... and unchanged in its solution) ; plus the stricture that “differences in the populations of the districts are allowed to satisfy imperatives in the general public interest” but cannot deviate from the average of the département by more than 20%.

But 20% over + 20% under = 50% difference because :

$$\begin{aligned} &\text{average} = 100\,000 \\ &\text{implies} \\ &80\,000 \text{ and } 120\,000 \text{ are tolerated, and} \\ &120\,000 \div 80\,000 = 1.5 \end{aligned}$$

(just as in Bavaria $15\% + 15\% = 35.3\%$).

Usually, also, there is a popular expectation of districts that are – in some sense – *compact*.

Model 1 :

Each canton is assigned a geographical *center*.

The computation :

- Chooses n district centers, and
- assigns every canton to exactly one center, thereby defining n districts.

Calculates each district's

- population : the sum of the populations of each of its cantons, and
- dispersion : the sum of the distances of each of its cantons from the district center weighted by the canton's population,

then chooses the centers and assigns the cantons to them so that

- district populations are within 3% of the ideal
- the sum of all district dispersions is a minimum.

Le Var : 43 cantons, 7 députés

Districts	Actual population	Computed population
1 ^e	73 946	122 247
2 ^e	86 693	128 574
3 ^e	144 595	122 593
4 ^e	143 492	131 507
5 ^e	132 397	128 127
6 ^e	180 153	132 951
7 ^e	137 165	132 397
inégalité	143.6%	8.8%

Mesure of inequality : $180\,153 \div 73\,946 = 2.436$

Model 3. Partisan-free districting

Idea : A member of the French Assemblée nationale (or of the US House of Representatives) represents his/her *single district* **and** his/her département (or State).

Accordingly :

The total vote of a party in a département (or State) determines the total number of its elected candidates (by the method of d'Hondt, to favor big political parties).

“The operations of *political* arithmetic aim at achieving useful research in the art of governing people ... One may with ease conceive that from such discoveries, and many others ..., obtained from calculations based on certain well founded experiments, a skilled minister would draw a mass of useful conclusions ... But often ministers (I dare not say without exception) believe they have no need to tire themselves with combinations and series of arithmetic operations: some imagine themselves gifted with a powerful natural genius that dispenses them from a course so long and painful ... Nevertheless, if the nature of the business demanded and permitted it, I have no doubt that we would convince ourselves that the political world, just as the physical world, may be regulated in many respects by weight, number and measure ”

Jean Antoine Nicolas Caritat de Condorcet

	Actual districts :		Calculated districts :	
	Number of députés	Inequality 1982	Inequality 1999	Inequality 1999
Hauts-de-Seine	13	44,17%	50,97%	18,41%
Val-d'Oise	9	32,87%	78,87%	17,70%
Alpes-Maritimes	9	26,63%	66,41%	18,94%
Var	7	31,66%	143,63%	8,76%
Puy-de-Dôme	6	24,53%	31,66%	1,72%
Vaucluse	4	17,85%	40,96%	5,71%
Tarn	4	25,11%	45,71%	1,69%
Mayenne	3	26,99%	28,82%	0,97%
Corrèze	3	18,47%	37,08%	0,01%
Meuse	2	28,20%	28,94%	0,17%
Hautes-Alpes	2	25,66%	30,64%	0,23%
Lozère	2	9,28%	13,85%	0,07%

Model 2 : “Set covering”

Canton	Possible district	“cost”	\min
#1	1	1	1
#2	1	1	1
#3	1	1	1
#67	1	1	1

Maryland : 8 seats – 2002 elections

Total vote Proportional share Seats by d'Hondt Actual seats				
Republican	752 911	3.634	4	2
Democratic	904 250	4.366	4	6

“Fair” allocations :

192 004	*88 954	75 721	34 890	60 758	147 825	49 172	*103 587
57 986	105 718	145 589	131 644	137 903	75 575	137 047	112 788

(Top line Republican, bottom line Democratic)

Michigan : 15 seats – 2002 elections

	Total vote	Proportional share	Seats by d'Hondt	Actual seats
Republican	1 474 178	7,417	7	9
Democratic	1 507 174	7,583	8	6

“Fair” allocations :

69254	156937	153131	149090	0	126936	121142	156525	141102	137339
150701	61749	61987	65950	158709	53793	78412	70920	*96856	77053
126050	61502	0	26544	48626					
*87402	140970	120869	145285	136518					

(Top line Republican, bottom line Democratic)

Nord-Ouest, European elections 1999.

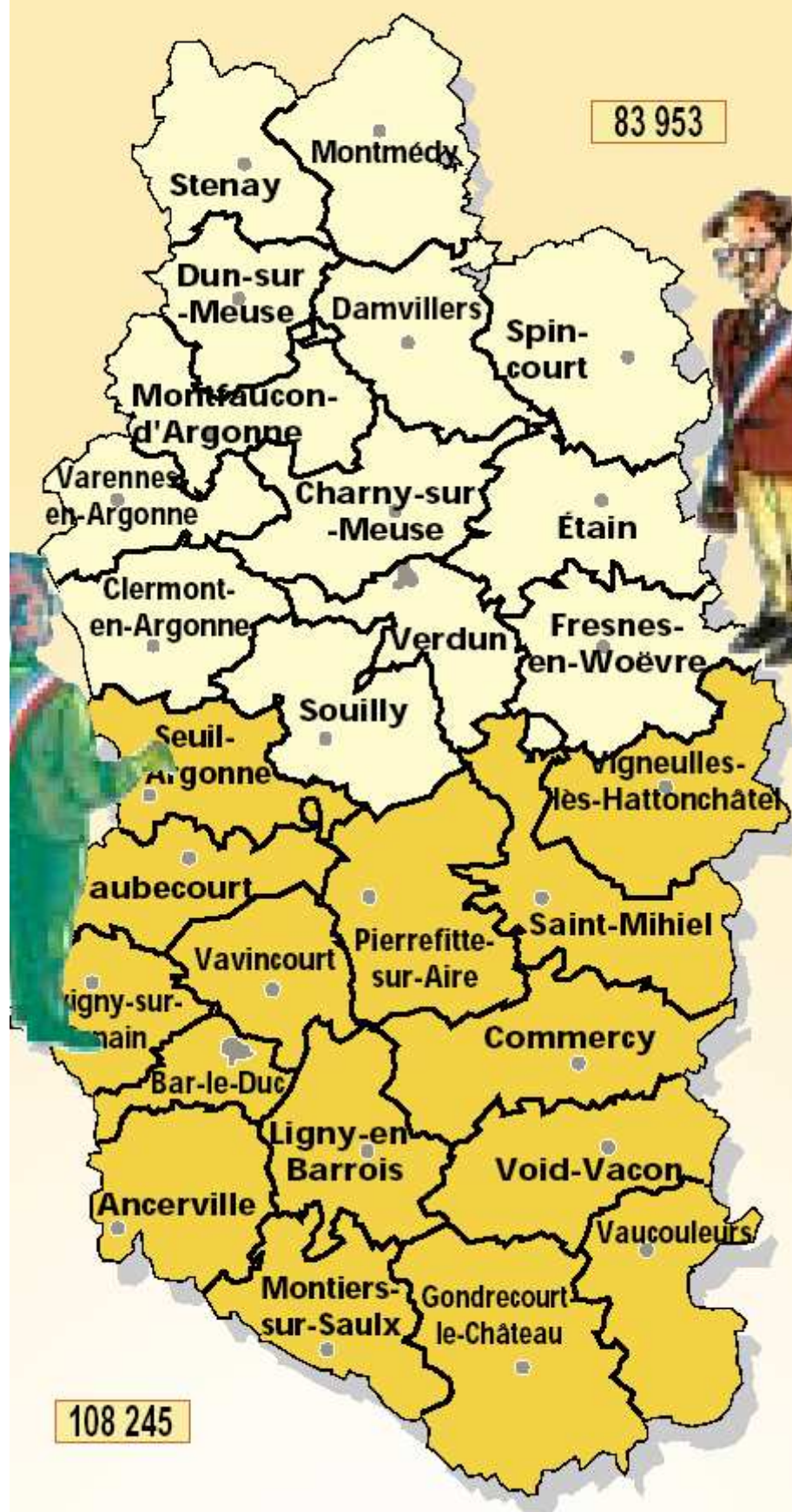
	UMP	Gauche	CPNT	ExD	Vert	PCF	UDF	ExG	Seats
Nord-Calais	260.199	270.363	107.135	126.035	93.337	113.743	88.929	83.333	5
Basse-Normandie	123.383	92.529	64.131	32.419	40.955	19.856	46.632	23.604	2
Haute-Normandie	129.159	116.964	36.173	53.566	48.711	43.467	42.842	34.182	2
Picardie	132.969	114.452	84.787	67.205	42.570	44.965	41.781	37.892	3
Seats due	3	3	1	1	1	1	1	1	12

	UMP	Gauche	CPNT	ExD	Vert	PCF	UDF	ExG	Total
Nord-Calais	1	1	1	1	1	1	1	1	8
Basse-Normandie	0	0	0	0	0	0	0	0	0
Haute-Normandie	1	1	0	0	0	0	0	0	2
Picardie	1	1	0	0	0	0	0	0	2
Seats due	3	3	1	1	1	1	1	1	12

divisor	UMP	Gauche	CPNT	ExD	Vert	PCF	UDF	ExG	Seats
1,865 Nord-Calais	139.517	144.967	57.445	67.579	50.047	60.988	47.683	44.683	5
0,916 Basse-Normandie	134.698	101.014	70.012	35.391	44.711	21.677	50.908	25.769	2
0,973 Haute-Normandie	132.743	120.210	37.177	55.052	50.063	44.673	44.031	35.131	2
1 Picardie	132.969	114.452	84.787	67.205	42.570	44.965	41.781	37.892	3
Seats due	3	3	1	1	1	1	1	1	12

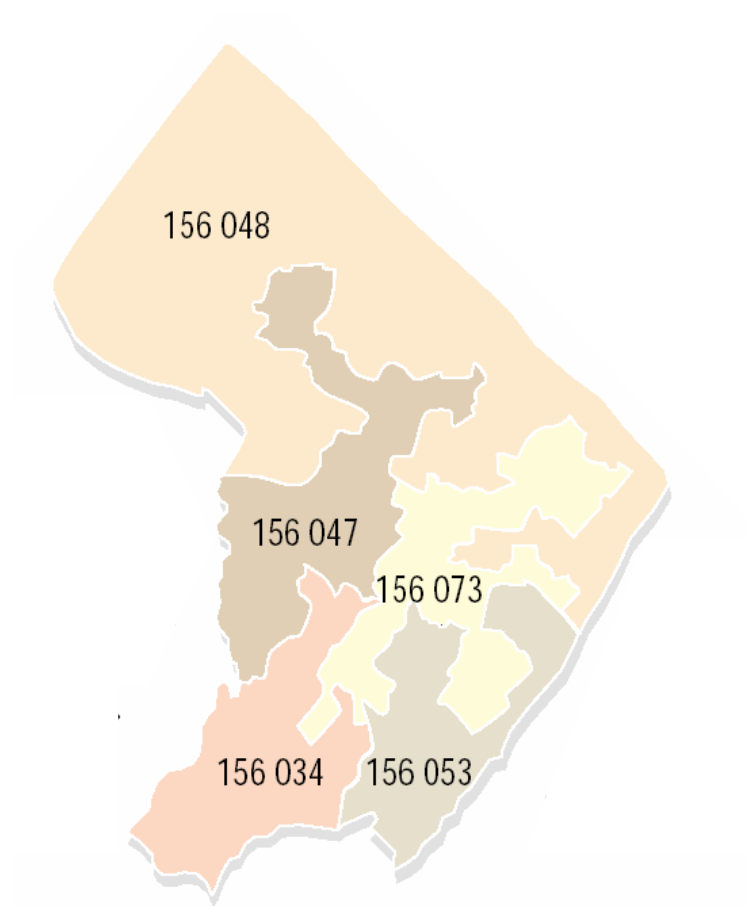
	UMP	Gauche	CPNT	ExD	Vert	PCF	UDF	ExG	Total
Nord-Calais	1	1	0	1	0	1	0	1	5
Basse-Normandie	1	0	0	0	0	0	1	0	2
Haute-Normandie	0	1	0	0	1	0	0	0	2
Picardie	1	1	1	0	0	0	0	0	3
Seats due	3	3	1	1	1	1	1	1	12

DÉCOUPAGE ACTUEL



DÉCOUPAGE CALCULÉ

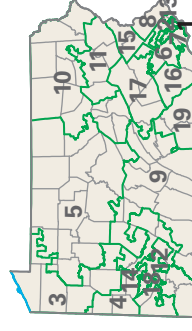
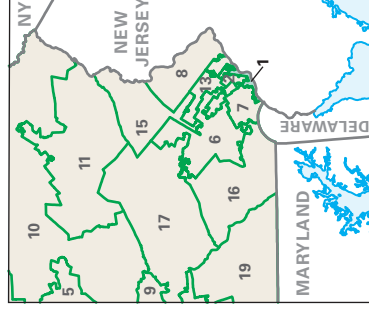
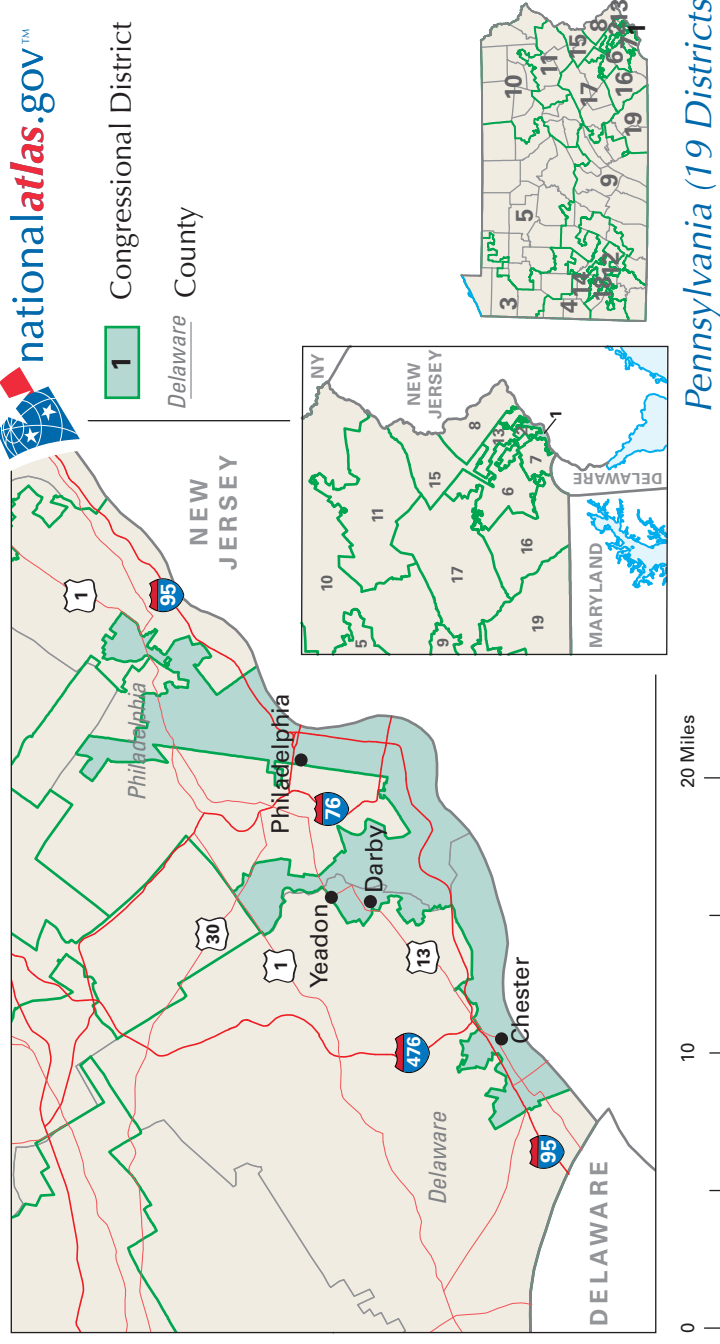




Bergen County (New Jersey), 1970.

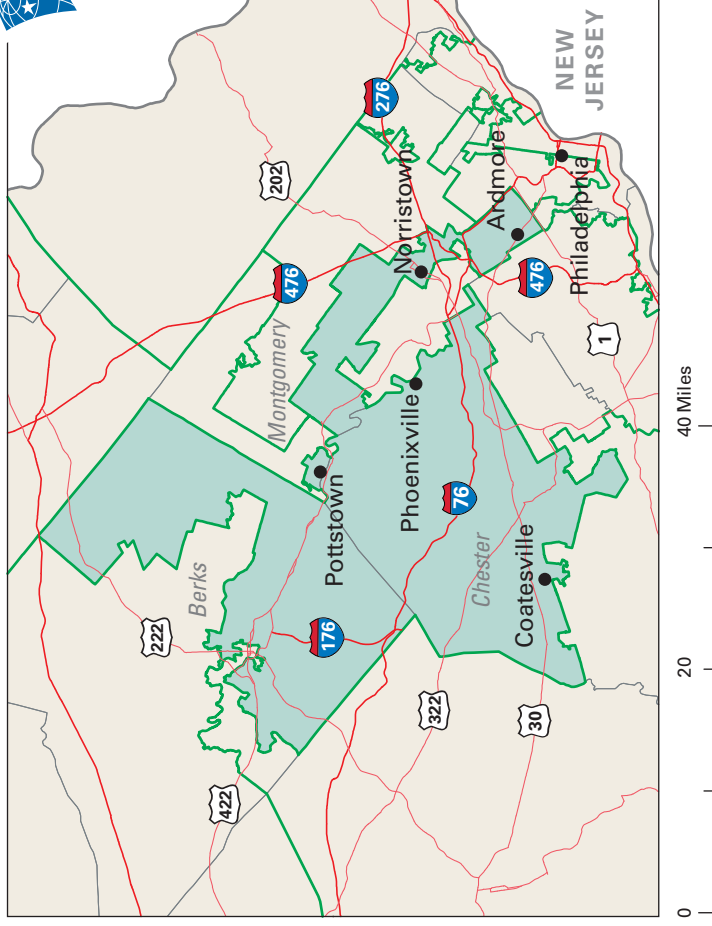
(67 municipalities partitioned into 5 connected districts with one goal : to minimize the differences in populations.)

Congressional District 1

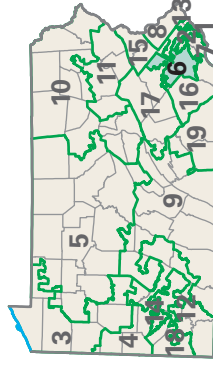


Pennsylvania (19 Districts)

Congressional District 6



6 Congressional District
Chester County



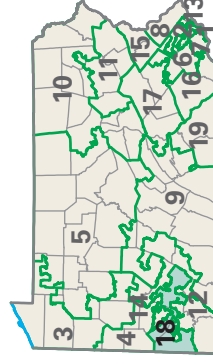
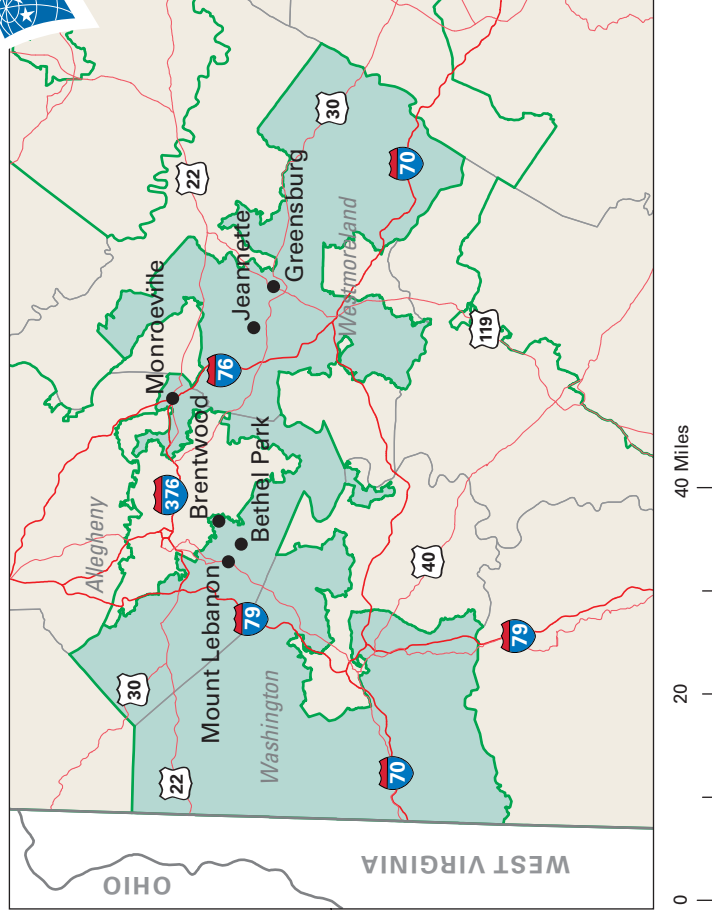
Pennsylvania (19 Districts)

Congressional District 18



nationalatlas.gov

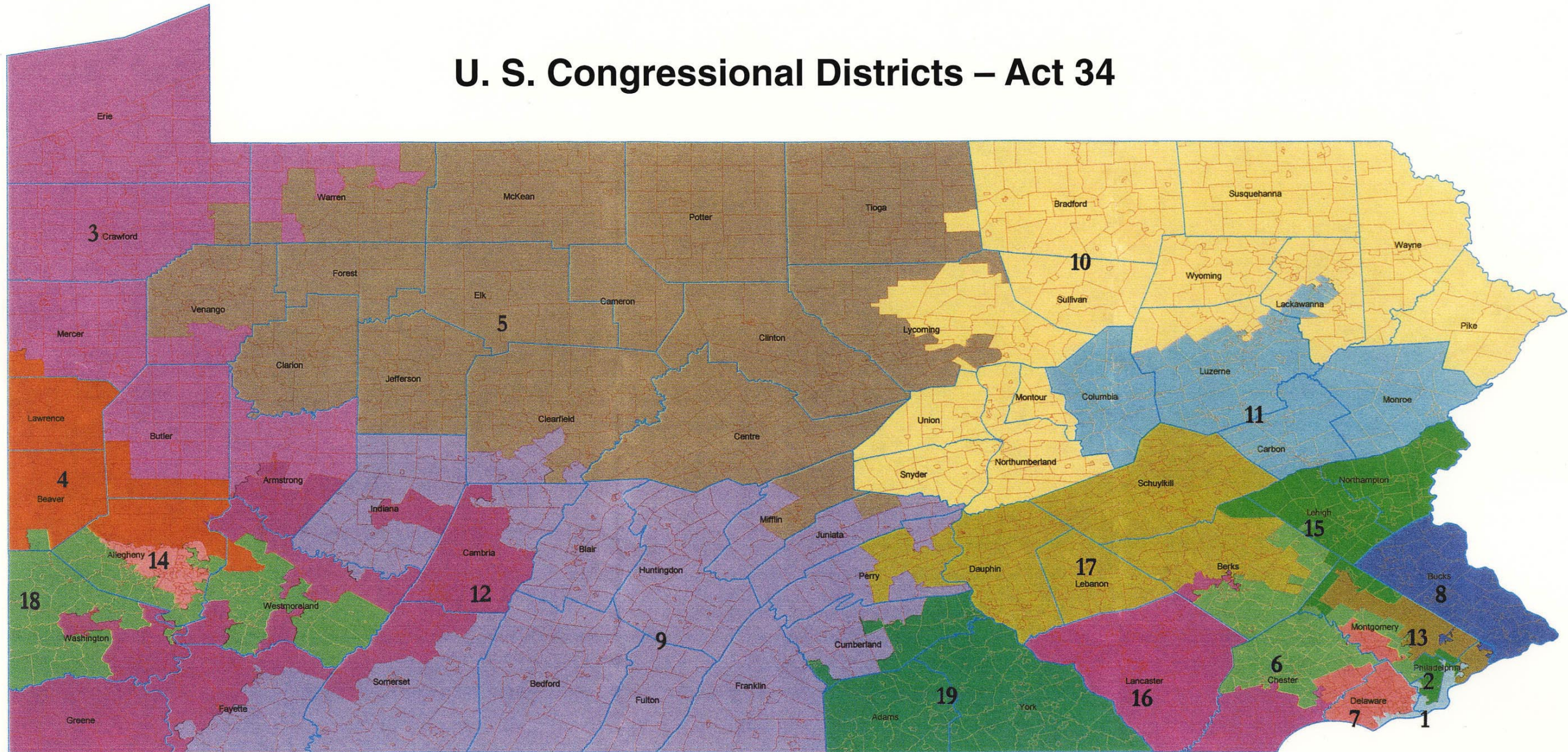
18 Congressional District
Washington County



Pennsylvania (19 Districts)

APPENDIX TO OPINION OF STEVENS, J.

U. S. Congressional Districts – Act 34



DÉCOUPAGE ACTUEL



DÉCOUPAGE CALCULÉ



CODE	CANTON	POPULATION
01	Aups	2 716
02	Barjols	8 105
03	Le Beausset	26 726
04	Besse/Isole	10 399
05	Brignoles	22 193
06	Callas	6 411
07	Collobrières	13 369
08	Comps/Artuby	1 109
09	Cotignac	7 354
10	Cuers	18 196
11	Draguignan	44 585
12	Fayence	18 127
13	Fréjus	50 536
14	Grimaud	28 643
15	Hyères-Est	36 304
16	Lorgues	15 805
17	Le Luc	18 621
18	Ollioules	39 003
19	Rians	9 776
20	Roquebrussanne	16 151
21	Saint-Maximin- Sainte-Baume	26 924
22	Saint-Tropez	19 753
23	Salernes	4 828
24	La Seyne/Mer	37 026
25	Sollies-Pont	26 065
26	Tavernes	3 436
27	Toulon 1er	11 005
28	Toulon 2ème	24 431
29	Toulon 3ème	26 910
30	Toulon 4ème	10 013
31	Toulon 5ème	10 136
32	Toulon 6ème	27 694
33	Toulon 7ème	9 647
34	Toulon 8ème	25 111
35	Toulon 9ème	15 692
36	La Crau	32 322
37	Le Muy	25 543
38	St-Mandrier/Mer	28 394
39	Saint-Raphael	30 671
40	Six-Fours-Plages	32 742
41	La Valette-du-Var	25 180
42	La Garde	24 365
43	Hyères-Ouest	26 424
	TOTAL	898 441



7. Juli 2004 FP/fp

Spektabiles,
sehr geehrter Herr Prorektor,
hohe Festversammlung,
liebe Damen Balinska Junior,
und – last but eigentlich first – verehrter Kollege Michel Balinski,

soon to become *Doctor rerum naturalium honoris causa Universitatis Augustanae*. Soon, *in spe*, it is my pleasure to emphasize, because Spektabiles Jungnickel will bestow the honorary degree on you only when, or shall I say: if, I have come to an end with my laudatio, which has barely even begun.

In fact, right away we are facing a certain problem. In case that of the many achievements of Professor Balinski there were too many or, mathematically speaking, infinitely many, my laudatio would never ever come to an end, and Professor Balinski would have to wait for the honorary degree infinitely long. Luckily, Michel Balinski distinguished himself in a subfield of mathematics that goes under the name of discrete mathematics, and that is characterized by dealing with finitely many items and finite sets. Hence, my dear Michel, I am confident that you will tolerate my saying that of your lasting achievements there are finitely many.

Since all good things come in threes, I shall concentrate, firstly, on your contributions to linear and to combinatorial optimization, secondly, on your contributions to discrete model building and, thirdly, on your contributions to the analysis of proportional representation systems. To begin with, however, I would like to introduce you to the audience as a human being, by telling a little bit from your vita, and to introduce us to you, by reviewing parts of the short history of our Institute.

Michel Balinski: Vita

Michel Balinski was born some seventy years ago, in Geneva, into a polyglott family. His father being occupied by his position as a Polish diplomat with the League of Nations, much of Michel's education laid in the hands of his grandparents. In fact, his grandfather was Ludwik Rajchman (1881-1965) who, then, was the Director of the League of Nations' Health Organization and, later, in December 1946, became the founder of Unicef, the United Nations International Children's Emergency Fund.

The Geneva start was followed by a few years in France which, fleeing from the invading German troops, ended in the exodus, via Lisbon, to the US. In the New World his mother was to continue to speak to him in French, thus helping the child to maintain the French roots.

Michel, having grown up outside New York City, spent most of his formative years and his academic career on the East coast. In 1954 he graduated from Williams College with a Bachelor's Degree in Mathematics, followed by a Master's Degree in Economics from the Massachusetts Institute of Technology and, in 1959, a PhD degree in Mathematics from Princeton University.

The academic affiliations of Michel Balinski include Princeton University, the Wharton School at the University of Pennsylvania, the City University of New York, Yale University, and the State University of New York at Stony Brook. The latter ran in parallel with a position of a Directeur de Recherche with the CNRS and the Laboratoire d'Econométrie of the Ecole Polytechnique. Over time the double appointment across the Atlantic proved too much of a burden. Hence in 1989 Michel Balinski moved permanently to the Ecole Polytechnique, and this is the position from which he retired to the status of emeritus, in 1999.

In the French system – other than in the German system, I enviously add – there is also a life after retirement, of which Michel Balinski has made and is still making good use, in his position as a Directeur de Recherche de classe exceptionnelle émérité. Judging from the rate of output of papers, attendance of conferences, and other academic activities I am happy to report that Professor Balinski must be alive and well.

In the forty years of professional career Michel Balinski distinguished himself with an exceptional number of successful activities beyond what a Professor normally is paid for, research and teaching. He founded the journal *Mathematical Programming*, which became a leading journal of the field. He served as president of professional societies, visited an impressive number of international universities as a guest lecturer, and served as the System and Decision Sciences Chairman of IIASA, the International Institute for Applied Systems Analysis in Laxemburg near Wien. In 1965 Balinski was awarded the prestigious Lanchester Prize of the Operations Research Society of America, which is a particular pleasure to mention since the Augsburg Mathematics Faculty includes with Karl-Heinz Borgwardt another Lanchester Prize winner.

Augsburg: Anwendungsorientierte Mathematik

Thus having innocuously found our way to Augsburg, let us stay here for a while. Founded by the Romans, the city of Augsburg looks back on more than 2000 years of history. In the middle ages, Augsburg was the premier banking place of the Old World, a place you would necessarily turn to, if you wanted to be elected emperor and were in need of a few thousands, or hundred thousands, of *Gulden* to bribe your electors. Strangely, though, the bourgeois elite of the Freie Reichsstadt never contemplated investing their money in a University.

We therefore have to admit that the University is less of a product of Augsburg's glorious past. Rather, founded in 1970, it is more of a political reaction to the post-68 syndrome. Which, on the other hand, makes the University a young lady, crispy and attractive.

When, in 1981, mathematics was added to the growing University, the challenge was to develop an image which would convey to the general public the idea of how modern, useful, and profitable mathematics is, both in the real world that we live in, as well as the complex world that we think in. The challenge was met by letting the Augsburg Mathematics Institute sail under the heading of *anwendungsorientierte Mathematik*, and this strategic orientation has proved to be extremely successful since.

It is difficult to properly translate *anwendungsorientierte Mathematik* into English. Applied mathematics would be too narrow, besides being already confined to the connotation of Angewandte Mathematik and, inappropriately, suggesting a contraposition with Pure Mathematics. I am afraid I cannot do better than translating *anwendungsorientierte Mathematik* rather literally into *application oriented mathematics* which, indeed, embraces abstract research such as pure mathematics, while at the same time prominently emphasizes the practical use to which mathematics is put.

Under the label *anwendungsorientierte Mathematik* a novel degree in *Wirtschaftsmathematik* was devised and was then, and still is today, the curriculum that most of our students choose to enroll in. In 1981 there was only a handful of mathematics departments at German universities offering such a degree. With the success saga spreading this has changed, and we have lost our unique position. The *anwendungsorientierte* concept carried not only beyond Augsburg to other campuses. On this campus, it also carried beyond mathematics to other fields. It provided an extremely fruitful starting point for the Augsburg Physics Department and, as of recently, of the Computer Science Department. The current strategic plan of the University provides some renewed visibility for the concept, by presenting it under the timely label of *innovative technologies*.

Ladies and gentlemen, I have reviewed part of the University history on this occasion, of conferring an honorary PhD degree on Michel Balinski, because he and his scientific œuvre testify in a prime way that the *anwendungsorientierte* interplay, of real world problems and complex academic solutions, is fascinating, fruitful, and never ending. As mentioned in the beginning, I will exemplify this claim by marking three of the fields where Balinski's scientific achievements stand out.

Contributions to linear and to combinatorial optimization

Balinski's 1959 Princeton PhD thesis, directed by Albert W. Tucker, was entitled *An Algorithm for Finding All Vertices of Convex Polyhedral Sets*. Mathematicians have always been interested in finding good algorithms, that is, descriptions which calculations have to be executed in order to solve a problem. After World War II, the advent of computers gave rise to a renewed interest in algorithms, and in particular those algorithms that are suitable for machine calculations.

The type of problems that are exceptionally well suited to be handled by a machine came to be known under the name of *linear programs*. Balinski's dissertation dealt with particular geometric structures that submit themselves to this approach, convex polyhedral sets (Vielecke), whose shape is determined by flat sides meeting in straight line edges, and edges meeting in vertices, *Eckpunkten*. Any such set can be described from an internal, primal point of view, or alternatively, from an external, dual view point. Just as we can describe this lecture hall by looking to the walls from the inside where we are now – which is the primal approach to the problem, or else by walking around on the outside and tell what we see then – the dual approach. At times the dual approach is quite persuasive, just think of the drinks that may be waiting outside. However, Balinski approached the problem more from an academic point of view and, together with his Doktorvater Al Tucker, published a long article on the *Duality Theory of Linear Programs* in the 1969 *SIAM Review*.

Another important subclass of problems are those, where the vertices of the convex bodies (konvexe Körper) have integer coordinates (ganzzahlige Koordinaten). Balinski was one of the first to extend the theory to such models. Nowadays there are many textbooks devoted to this type of problem, but in 1965 there was none. The subject was new, and Balinski's seventy-page 1965 overview article served as a welcome reference and first textbook for the new field. The article enjoyed the fate, rather rare in mathematics, of being reprinted twice, in 1968 and in 1970.

Another subset of combinatorial results comes under the inviting name of *stable marriage theorems* (stabile Heiratssätze), proving by terminology better than by anything else that mathematics is so utterly anwendungsorientiert. The problem is standard. There is a set of ladies and a set of men. Each lady likes certain of the men and has her preferences among them, but detest the others. Similarly, each men has his preferences among the women he likes, but cares not a whit about the others. The mathematical question is this: Can men and women be happily married given their respective preferences?

Imagine what could go wrong if Monika and Friedrich, say, were married to others, and yet, Monika preferred him to her current mate, while at the same time Friedrich preferred Monika over his current mate. They would be unhappy and, provided the two couples meet too often, they would abandon their current mates for each other. Though I grant that marriage is a very serious business, it is a peculiar trait of mathematicians to go to the simplest possible situation that reveals the essence of a problem, however frivolous it may sound. There are also polyandrous versions of the question, where every lady may have several husbands, and polygamous versions, starring men with many wives.

Are there marriages which would avoid the sad instability of our example? The mathematical theorem answers: Yes, there always are! Of course, the mathematical results are independent of the narrative frame in which they are presented. There are important practical problems submitting themselves to just the same analysis: admitting students to universities, appointing candidates to jobs, or allocating hours of work on different tasks to workers of different qualifications. All these instances come under what now is called “two-sided markets”, for which the mathematical approach provides a vital aid how to match, allocate, or apportion resources. As with other problems, mathematics focusses on two issues: Can it be done at all and, if so, how to do it efficiently. Balinski and co-authors have contributed considerably to the realm of marriage theorems and, in a recent 2003 *American Mathematical Monthly* note exemplify its usefulness for *Admissions and Recruitment*.

Contributions to discrete model building

Problems of matching and allocation, as just outlined, are categorized as *discrete mathematics*. The attribute “discrete” is used for the very reason why we term a person discrete, as somebody who honors the individual, who acknowledges that there are features peculiar to an individual rather than being shared by many, as somebody who refrains from generalizing inappropriately. In social life the opposite is indiscretion. In mathematics, however, the opposite of discrete mathematics is continuous mathematics.

Discrete mathematics counts items; it rules out continuous transitions from one to the other. Each candidate stands for herself or himself, each institution is recognized as a unit on its own, each seat in parliament is honored as a valuable entity by itself. Such constructs as fractional candidates, or fractional institutions, or fractional seats in parliament are meaningless. The friction between mathematical terms and practical needs becomes particularly apparent in consulting, when the – mathematical – consultant wants to persuade the – non-mathematical – consultee of the usefulness of the approach. While model building itself may be more of an art than a science, the mathematical problems thereby generated are abundant, and challenging.

Michel Balinski’s experience to apply abstract mathematical concepts to concrete problems of economics and decision making is based on an impressive experience as a mathematical consultant, for such companies as de Borden Mills Inc., the RAND Corporation, Mathematica Inc., Mobile Oil Research Laboratories, ORTF, Econ Inc., and others. His pointed opinions on where mathematics can contribute to the solution of practical problems is all too noticeable throughout his technical papers, be it matching problems as covered by the type of marriage theorems mentioned above, or problems of assignment, allocation, or apportionment. As a third group of examples, I would like to finally turn to his work on apportionment methods for proportional representation.

Contributions to the analysis of proportional representation systems

Balinski's research into the mathematics of apportionment and proportional representation originates from the early 70's, much of it together with his junior co-author Peyton Young. The collaboration of the two culminated in the 1982 monograph on *Fair Representation: Meeting the Ideal of One Man, One Vote*. The first edition from Yale University Press was followed by a 1987 Japanese translation and, in 2001, by a second edition whose pagination is identical with that of the first edition. The book centers around the apportionment problem as it manifests itself for the US-American House of Representatives. There the issue is to apportion (zuteilen) the 435 house seats among the 50 States of the Union, proportionally to the population counts of the decennial census data.

Of course, parliamentary seats are in the first place more of a political than of a mathematical nature. However, when one of the seats was contested in 1992, the United States Supreme Court found it appropriate to include in the decision a six-page review of the Balinski/Young monograph. I would bet that theirs is the only mathematics book that ever got read and reviewed by any supreme court throughout the world. The judges, while not questioning the mathematical correctness of the book, did improve on its political correctness, by turning the untimely Balinski/Young motto of “one man, one vote” into the more equal principle of “one person, one vote”. It may have escaped their attention that, in mathematics and statistics, the data unit “man” refers to what in the old days the great Latin writers would have worded as “homo”, and not as “vir”. As down-to-earth scientists we would never set up a theory that visibly excludes half of mankind, or politically more correct, half of personkind.

The Balinski/Young *Fair representation* work is actually two books in one. The first half carefully reviews the historical experience that accumulated over more than 200 years of US history. I find these first hundred pages a gem of scientific writing, always lucid, occasionally thrilling, and at times entertaining. The authors aim, and succeed, in extracting from the historical experience general rules which, in mathematical language, may serve as an axiomatic foundation of a theory of apportionment, which then is laid out in the second half of the book.

For instance, one such axiom demands that changes in representation agree with changes in population. If, relative to several competing groups, one is growing larger, then the number of its representatives should increase. But does it? Not in the system that we use for the election of the Deutschen Bundestag where, weird as it may sound, more Zweitstimmen votes for a party may result in fewer Bundestag seats.

Another axiom stipulates that a parliamentary body that grows in size will never see a party shrink in its number of representatives. But does it? Not in Germany. When, in 1989, the steering committee of the Wetteraukreis in Hessen was formed, the two major parties raised the number of seats from nine to ten. Why? Because this made one of the minor parties drop from one representative to none. That is, not only was a new seat created by enlarging the committee size from nine to ten, but, due to the peculiarities of the apportionment method used, an old seat was taken away from an unwanted competitor who was thereby pushed out. Not surprisingly, the two seats thus generated benefitted the two major parties.

Laying down general principles, or axioms as we say in mathematics, enables us to classify the many methods that are available to convert population or vote counts into numbers of representatives. These mathematical classifications are carried out in the second half of the Balinski/Young monograph, where the authors scrutinize the apportionment methods that are in use all over the world against the general axioms that appear so compelling and hard to deny.

Michel Balinski pursued the subject into many different directions. One of them includes biproportional representation methods, wherein proportionality is achieved in two directions, one along the regional subdivision of the population, the other, along party lines. I had the privilege of proposing one such biproportional method to the Kantonsrat Zürich who, in fact, adopted it almost unanimously to include it in their new electoral law.

Another line of Balinski's research aims at the districting problem, that is, achieving electoral equality in electoral districts (Wahlkreise, Stimmkreise). We will hear more about this problem in a minute or two from Professor Balinski himself.

I have subsumed Michel Balinski's research work under the Augsburg standard of Anwendungsorientiertheit, and I would like to end by drawing your attention to another point of what Anwendungsorientiertheit includes. Namely, proliferating the mathematical findings, not only as technical papers in academic journals, but also as nontechnical articles in the public science press. Who otherwise would publicize the findings, if not those who generate them? You will not be surprised to hear that Michel Balinski has done so, in *Le Monde*, *Pour la science*, *Spektrum der Wissenschaft* and other press products.

Naturally it always remains a challenge to translate dry academic truths into juicy public stories, and a welcome trick is to embellish the presentation by drawing on quotes from literary and intellectual authorities. If my laudatio were in German I certainly would have taken recourse to Goethe and Schiller. As it is in English, I have to switch to the British G&S-counterpart, Gilbert and Sullivan, and end with a three-liner quoted by Professor Balinski in order to dismiss any doubt that his proposed mechanism for a stable marriage assignment is optimal. As for me, the quote is to imply that the mechanism of the Mathematisch-Naturwissenschaftliche Fakultät der Universität Augsburg to nominate their PhD laureates is – doubtlessly – equally optimal:

*Of that there is no matter of doubt–
No probable, possible, shadow of doubt–
No possible doubt whatever.*
(Gilbert and Sullivan, *The Gondoliers*)